on freezing. The gases are presumed to be sufficiently pure to exclude significant systematic errors due to contaminants.

The large difference between the volume change of argon during freezing found in these experiments and the volume change reported by Bridgman (2) (Figure 4) cannot be ignored. There are differences ranging from 35 to $100 \%$ between Bridgman's results and those of the authors. Any possible error in the piston-displacement method used in the present work would not seem large enough to account for this discrepancy. "Squaring-up" the experimental curve of displacement us. pressure could lead to slightly too high a value, although no difficulty was found in reproducing the published data on the change in volume of water during freezing at comparable pressures. An undiscovered leak of gas prior to measuring the argon sample volume after the experiment, and an equivalent error in computing the sample size at the start of the experiment (from the vessel size and published PVT data on argon) would be an unlikely coincidence.

Bridgman pointed out that his bulb-filling method could be in error on the low side if the filling capillary became
plugged with solid. This might possibly explain why Bridgman reported lower values than these.

## ACKNOWLEDGMENT

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## Velocity of Sound in

## Compressed Gases

T. K. SHERWOOD<br>Massachusetts Institute of Technology, Cambridge, Mass. $\mathrm{V}_{\text {ALUES }}$ of the velocity of sound in compressed gases are required for various technical purposes, as for the design of pulsation dampeners and the calculation of throat velocities in supersonic nozzles. Few data are available and the sound velocity must usually be calculated from other properties of the gas. Simple formulas are available for ideal gases, which serve adequately for many purposes, but these are not useful if the gas properties differ appreciably from ideality.

Reliable values of the sound velocity may be obtained if the low-pressure heat capacity and an accurate equation of state for the gas in question are known. The present study explores the possibility that useful values of sound velocity might be calculated from a general reduced equation of state for all gases. The results are compared with limited experimental data on $\mathrm{CH}_{4}, \mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{6}, \mathrm{C}_{3} \mathrm{H}_{8}$, i- $\mathrm{C}_{5} \mathrm{H}_{12}$, $n-\mathrm{C}_{3} \mathrm{H}_{12}, \mathrm{He}$, and $\mathrm{CO}_{2}$.

Thermodynamics provides the following relations:

$$
\begin{align*}
C^{2} & =-a v^{2}\left(\frac{\partial P}{\partial v}\right)_{s}=\frac{a R T K Z^{2}}{Z-P_{r}\left(\frac{\partial Z}{\partial P_{r}}\right)_{T_{r}}}  \tag{1}\\
K & =\frac{C_{p}}{C_{r}}=\frac{C_{p}{ }^{\circ}+\Delta C_{p}}{\left(C_{p}{ }^{\circ}+\Delta C_{p}\right)-\left(C_{p}-C_{r}\right)}  \tag{2}\\
\Delta C_{\rho} & =\frac{\partial}{\partial T_{r}}\left(\frac{H-H^{0}}{T_{r}}\right)_{P_{r}}  \tag{3}\\
C_{P}-C_{v} & =\frac{R\left[Z+T_{r}\left(\frac{\partial Z}{\partial T_{r}}\right)_{P_{r}}\right]^{2}}{Z-P_{r}\left(\frac{\partial Z}{\partial P_{r}}\right)_{T_{r}}} \tag{4}
\end{align*}
$$

Here $C$ is the sound velocity and $Z$ is the compressibility

For estimating sound velocities, the tables in this article give greater accuracy than graphs. The tables are also practical for other purposes-for example, data on heat capacity function have been published by Edmister and by Lyderson, Greenkorn, and Hougen, based on earlier generalized equations of state. The present data, based on Pitzer's recent work, are calculated by numerical differentiation.
factor, which is assumed to vary only with $P_{r}$ and $T_{r}$, the reduced pressure and the reduced temperature, respectively. The above relations are the same as those used by Buthod and Tien (1) in a recent similar study.
Of the several reduced equations of state available, that of Pitzer and Curl (12) was selected, values of $Z$ being taken from their tables for $Z_{c}=0.27$ (this is equivalent to an "acentric factor," $\omega$, of 0.2625 , since $Z_{c}=0.291-0.08 \omega$ ). Values of $\partial Z / \partial P$, and $\partial Z / \partial T$, were obtained from large graphs of $Z$ us. $P_{r}$ or $T_{r}$, using numerical or graphical methods, as seemed appropriate. Values of $\Delta C_{p}$ were obtained in the same manner, using Pitzer and Curl's values of $\left(H-H^{0}\right) / T_{c}$.

The result of these calculations is three functions:

$$
\begin{aligned}
C^{2} / a R T K=f_{1}\left(P_{r}, T_{r}\right) & \text { (Table I) } \\
\Delta C_{p}=f_{2}\left(P_{r}, T_{r}\right) & \text { (Table II) } \\
C_{P}-C_{v}=f_{3}\left(P_{r}, T_{r}\right) & \text { (Table III) }
\end{aligned}
$$

Values of $C_{p}^{0}$ needed in Equation 2 are available from API Research Project $44(2,14)$.

Using these tables, values of $C$ are calculated as follows: $P_{r}$ and $T_{r}$ are obtained as ratios of the specified temperature and pressure to the known critical properties $\left\langle\right.$ if $P_{c}$ and $T_{c}$ are not known, they may be estimated closely by the methods described by Reid and Sherwood (13) ]; $\Delta C_{p}$ and $C_{p}-C_{v}$ are found from Tables II and III and $K$ is calculated from $C_{p}^{0}$ using Equation 2; $C^{2} / a R T K$ is obtained from Table I, and $C$ calculated by substitution.
The procedure will be illustrated by estimating $C$ for ethane at $90^{\circ} \mathrm{F}$. and 116 atm . From the literature, $P_{c}=48.3$ $\mathrm{atm} ., T_{c}=550^{\circ} \mathrm{R}$., and $C_{p}^{0}=12.8$; whence $P_{r}=2.4$ and $T_{r}=$ 1.00. From Tables II and III $\Delta C_{p}=16.53, C_{p}-C_{v}=15.93$,


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and from Equation $2 K$ is 2.23 ．From Table I，$C^{2} / a R T K$ is 2．316；whence：
$C^{2}=\frac{2.316 \times 4615 \times 550 \times 2.23}{30}=437,000 ; C=660$ meters per second
The experimental value is about 583 meters per second．
Figure 1 compares values calculated in this way with data obtained from various sources．Although the result is fair， it is evident that the method of calculation may lead to serious error．There are several possible reasons for the discrepancies：

1．Equation 1 is for sound at＂low＂frequencies in a gas under equilibrium conditions of molecular excitation．This should not be important，since the data quoted were obtained at frequencies from 3 mc ．to 900 kc ．per second （11）．

2．The generalized reduced equation of state is not suff－ ciently accurate，and the use of values based on a single $Z_{c}$ introduces additional errors when applied to various gases． Values of $Z$ are probably correct to within a few per cent， but the derivatives $\partial Z / \partial T_{r}$ and $\partial Z / \partial P_{r}$ are subject to large error in the vicinity of the critical．This is particularly true for $\Delta C_{P}$ ，which is obtained by two differentiations of $P-V-T$ data．

3．Accurate measurement of slopes of curves is difficult． This seems to have introduced no great error，except in the immediate vicinity of the critical．
4．There are experimental errors in the values of $C$ ．There is no reason to believe that this was important；it was necessary，however，to obtain most of the data by reading values from small graphs given in place of tables in the published reports．

At the critical temperature，$C$ decreases steadily as pres－ sure is increased above atmospheric，passing through a sharply pointed minimum at the critical pressure．The method of calculation described is evidently incapable of defining this sharp dip with reasonable accuracy．Above the critical pressure $C$ rises sharply and at high values of $P_{r}$ reaches values which are large relative to $C$ at 1 atm ．The sharp dip at $P_{r}=1$ becomes less and less pronounced as $T$ ，is increased from unity．

In reviewing the calculations，it appears that discrep－ ancies of more than $10 \%$ between calculated and observed values of $C$ were encountered in the following regions： for $\mathrm{CH}_{4}\left(Z_{c}=0.289\right)$ ，at $P_{r}$ greater than 6 （about 4000 p．s．i．g．）；for $\mathrm{C}_{2} \mathrm{H}_{4}$（near the critical temperature）at $P_{r}$ between 1.0 and 2．0；for $\mathrm{C}_{2} \mathrm{H}_{6}$（ at $T_{r}=1.00$ ）from $P_{r} 1.8$ to 2．2；for $\mathrm{C}_{3} \mathrm{H}_{8}$ ，from $P, 0.8$ to 3 at $T_{r}=0.95,0.8$ to 2 at $T_{r}=1.00, P_{r}=1.6$ to 2.2 at $T_{r}=1.15, P_{r} 1.6$ to 1.8 at $T_{r}=1.20$ ，and $P_{r} 4$ to 5 at $T_{r}=1.3$ ；for $\mathrm{C}_{5} \mathrm{H}_{12}$ from $P_{r} 0.8$ to 1.4 at the critical temperature；for helium（well above the critical temperature）at pressures higher than $P_{r}$ of 6.0 （ 4200 p．s．i．g．）．The calculated values for $\mathrm{CO}_{2}$ do not agree well with the data of Herget（3）for $T_{r}$ close to unity in the range of $P_{r}$ from about 0.5 to 1．5．

Many of the data were obtained in the critical region， and comparison of data with calculated values presents a severe test of the calculation procedure．Even more refined calculation procedures fail in the critical region．For example，the values of $C$ calculated by Hilsenrath and others（4）for $\mathrm{CO}_{2}$ ，using an equation of state designed especially for this gas，are less than $5 \%$ of the experimental value at $38^{\circ} \mathrm{C}$ ．（ $T_{r}=1.024$ ）and 100 atm ．The calculated values based on the generalized reduced equation of state check Herget＇s data at $38^{\circ} \mathrm{C}$ ．within $10 \%$ except in the range of $P_{r}$ from about 0.7 to 1．4．

Evidently the method presented is sufficiently good for engineering purposes for most common gases at tempera－ tures greater than $T_{r}=$ about 1．3．The pressure region of greatest discrepancy moves from near the critical at the critical temperature to values of $P_{r}$ of 6 or more as $T_{r}$ is increased to 3.


Figure 1. Velocity of sound in compressed gases

## ACKNOWLEDGMENT

Thanks are due Sheldon Dean and H.L. Gabelnick, who prepared Tables I, II, and III and made the calculations on which comparisons with experimental data are based.

## NOMENCLATURE

[^0]$(\partial P / \partial v)_{,}=$rate of change of pressure with volume at constant entropy

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[^0]:    $a=$ dimensional constant; $a R=(4615 / M) \mathrm{sq} . \mathrm{m} . /(\mathrm{sec}).\left({ }^{\circ} \mathrm{R}.\right)$
    $C=$ velocity of sound, meters per sec.
    $C_{p}=$ heat capacity at constant pressure, B.t.u./(lb. mole) ( ${ }^{\circ}$ F.)
    $C_{p}^{0}=C_{p}$ at "low pressure" (in ideal gas range)
    $C_{v}=$ heat capacity at constant volume, B.t.u. $/\left(\mathrm{lb}\right.$. mole) ( ${ }^{\circ}$ F.)
    $H=$ enthalpy, B.t.u./lb. mole
    $H^{0}=$ enthalpy at low pressure (ideal gas range), B.t.u./lb. mole
    $K=$ ratio of heat capacities, $C_{p} / C_{s}$
    $M=$ molecular weight
    $P=$ pressure, atm
    $P_{c}=$ critical pressure, atm.
    $P_{r}=$ reduced pressure, $=P / P_{c}$
    $R=$ gas constant, B.t.u./(lb. mole) $\left({ }^{\circ}\right.$ R.)
    $T=$ temperature, ${ }^{\circ} \mathrm{R}$.
    $T_{c}=$ critical temperature, ${ }^{\circ} \mathrm{R}$.
    $T_{r}=$ reduced temperature, $=T / T_{c}$
    $=$ specific volume, cu. ft./lb.
    $V=$ molal volume, cu. ft. $/ \mathrm{lb}$. mole
    $Z=P V / R T$
    $Z_{\mathrm{c}}=$ value of $Z$ at $T_{c}$ and $P_{c}$
    $\Delta C_{p}=C_{p}-C_{p}^{0}$

